Radio Science Techniques for Solar System
Tests of General Relativity

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Summary

Scientists utilize radio links between spacecraft and Earth or between spacecraft to examine changes in the phase/frequency, amplitude, line-width, and polarization, as well as round-trip light time of radio signals to investigate geophysical phenomena and for tests of fundamental physics including the theory of General Relativity. The BepiColombo Mercury Orbiter Radio-science Experiment (MORE) team will carry out high precision tests of relativistic gravity in the most desirable “laboratory” in the solar system, the gravitational field of the Sun. Being the innermost planet, Mercury is the ideal test mass for probing general relativity. Range and range-rate measurements from radio tracking a spacecraft in orbit around Mercury, with frequent superior solar conjunctions, provides abundant occasions to explore relativistic gravitational effects of the sun in addition to the structure of the solar corona. Figure 1 illustrates an example of the Radio Science investigations with BepiColombo. Figure 2 illustrates the MORE end-to-end instrumentation with two uplink radio signals transmitted simultaneously from a ground station and three coherent downlink signals are coherently returned back by the spacecraft.

Background

General relativity, Einstein’s theory of gravity, has passed every test to date. The incompatibility of general relativity and quantum mechanics, however, has led scientists to question the ranges of their validity and to believe that either one or both will ultimately fail. Furthermore, cosmological observations that the universe undergoes phases of accelerated expansion provide compelling motivations to seek more accurate laws of gravity. The theory of general relativity will likely require a modification such as the inclusion of a scalar term in the field equations. Deviation of the values of the parameterized post-Newtonian (PPN) parameters from those expected for general relativity at the level of $10^{-7}$ to $10^{-5}$ are predicted, as discussed in Damour and Nordtvedt (1993). Experimental detection of violations of the theory would have significant implications in physics and cosmology. Such experiments require technological advances which have been slow and often very costly, with incremental improvements typically made on the time scale of a decade.

A unique opportunity is available via the European Space Agency’s Mercury Planetary Orbiter (MPO), one of two spacecraft comprising the multi-national BepiColombo mission to Mercury; the second spacecraft is a Japanese Mercury Magnetospheric Orbiter. ESA selected the MORE team for the MPO to investigate relevant PPN parameters, solar oblateness, and possible time variation of the gravitational constant in addition to planetary geophysical objectives. To reach orbit in 2019, MPO is specifically designed for Radio Science observations with flight instruments contributed by the Italian Space Agency. Compatible Radio Science ground instrumentation is proposed to be provided by NASA via the Deep Space Network.

Figure 1: An illustration of the relativistic bending of a radio beam transmitted by the Mercury Planetary Orbiter and received on Earth.
Scientific Goals and Objectives

MORE’s scientific goals are to carry out high precision dynamic tests of relativistic gravity in an ideal laboratory as well as characterize the structure of the solar wind in and out of the solar ecliptic. The conventional framework for discussing solar system tests is the post-Newtonian parameterization. General relativity predicts definite values of the parameters but alternate theories of gravity predict deviations from these values. Nearly every metric theory of gravity can fit into the generalized 10-parameter PPN framework except for possible cosmological effects on the gravitational constant (Ashby et al., 2007). Of the 10 parameters, 4 are considered for improvement by MORE techniques, namely the PPN parameters $\gamma$, $\beta$, $\eta$, and $\alpha_1$. In addition, the solar oblateness will be determined with much improved accuracy, useful information will be obtained on the possible rate of change of the gravitational constant, and properties of the solar corona will be monitored accurately. Thus the objectives of the MORE investigation are:

**Determine $\gamma$ to an accuracy of $2 \times 10^{-6}$:**
In the PPN formalism according to Shapiro (1967), Will (1971, 1993) and others, $\gamma$ is a measure of how much space curvature is produced by a unit rest mass. The theory of general relativity, where $\gamma = 1$, predicts that a ray of light grazing the Sun is deflected by 1.75 arcsec and delayed in time by roughly 200 microseconds. Deflection and delay experiments place constraints on $\gamma$. The best accuracy measured to date for this parameter is $2.3 \times 10^{-5}$ from the Cassini experiment (Bertotti et al., 2003). MORE will achieve an accuracy for $\gamma$ of $2 \times 10^{-6}$, an improvement of one order of magnitude.

**Determine $\beta$ to an accuracy of $\sim 3 \times 10^{-5}$:**
In the same formalism, $\beta = 1$ for general relativity and is a measure of the nonlinearity in the superposition law for gravity. MORE will achieve an accuracy for $\beta$ of $\sim 3 \times 10^{-5}$ or better, which is significantly better than the best current accuracy of $1.2 \times 10^{-4}$ (Williams et al., 2004) which was derived by determining $\eta$ and using the Cassini value for $\gamma$ in the $\eta=4\beta-\gamma$3 parameter relationship.

**Determine $\eta$ to an accuracy of at least $4.4 \times 10^{-4}$:**
If gravity is described by a metric theory, the PPN parameter $\eta$ is a linear combination of the parameters $\gamma$ and $\beta$ and equals $4\beta-\gamma$3 in the Nordtvedt relationship. It addresses the difference between radial and transverse stress of gravity and bears on possible strong equivalence principle violation. MORE will achieve an accuracy for $\eta$ of $4.4 \times 10^{-4}$ (Ashby et al., 2007) to possibly $2 \times 10^{-5}$ (Milani et al., 2002), comparable with or an improvement over the current accuracy of $5 \times 10^{-4}$ (Williams et al., 2004).

**Determine $\alpha_1$ to an accuracy of $7.8 \times 10^{-6}$:**
According to Will (2006), while the parameters $\gamma$ and $\beta$ are used to describe “classical” tests of general relativity and are, in some sense, the most important, they are the only non-zero parameters in general relativity and scalar-tensor gravity. The parameters $\alpha_1$, $\alpha_2$, and $\alpha_3$ measure whether or not the theory predicts post-Newtonian preferred-frame effects. The MORE simulations by Milani et al. (2002) achieve a determination for the PPN preferred-frame parameter $\alpha_1$ to an accuracy of $7.8 \times 10^{-6}$, a significant improvement of the present accuracy of $\sim 10^{-4}$ (Will, 2006).

**Determine the solar oblateness to an accuracy of $4.8 \times 10^{-9}$:**
Measurement of the relativistic PPN parameter (such as $\beta$) is inextricably connected with the solar quadrupole moment $J_2$, which contributes, just as the relativistic corrections do, to the advance of Mercury’s perihelion. However, for solar studies, it is reasonable to also consider the accuracy with...
which \( J_2 \) can be determined if general relativity is assumed to be correct. The expected accuracy for \( J_2 \) will provide information about the differential rotation of the solar core and will be relevant to better understanding of the structure of the deep interior of the Sun. MORE will determine the solar oblateness in a dynamical measurement to an accuracy of \( 4.8 \times 10^{-9} \), much more accurate than the present estimate of \( 4 \times 10^{-8} \) (Milani et al., 2002).

**Test any time variation of the gravitational “constant,” \( G \), to an accuracy of \( 3 \times 10^{-13} \) per year:**
Alternate theories of gravity include cosmologically evolving scalar fields that lead to time variability of fundamental physical constants such as the gravitational constant, \( G \). Lunar laser ranging experiments have placed limits on variation of \( \dot{G}/G = (4 \pm 9) \times 10^{-13} \) per year (Williams et al., 2004), making the uncertainty \( 9 \times 10^{-13} \). Before this result was obtained, Will (1993) summarized recent classical tests of post-Newtonian gravity and the improved observational constraints on time variation of the gravitational constant first by ranging measurements to the Viking spacecraft at Mars, lunar laser ranging measurements, and pulsar timing data and quotes a suggestion by Bender et al., (1989) of the possibility of reaching an accuracy of the order of \( 10^{-14} \) per year for a Mercury orbiter with ranging accuracy of the order of \( 20 \) cm. MORE will improve on the constraints for detecting a variation of this fundamental “constant” to an accuracy of \( 3 \times 10^{-13} \) per year from one year of measurements, and will provide an independent check of the lunar laser ranging result; after 2 years, the accuracy would improve to \( 1 \times 10^{-13} \) per year.

**Characterize the solar corona:**
During the relativity observations, the Earth-spacecraft line-of-sight necessarily passes close to the Sun. As the radio beams propagate through the solar corona, information about the near-Sun plasma is imposed on the radio signals. These radio-wave propagation effects are noise for the MORE relativity observations and will be estimated and largely removed using the sophisticated MORE radio system (spacecraft and DSN components). To implement the required calibration, however, the plasma effects will be known with excellent accuracy. This presents an opportunity to use the radio-wave scintillations for science: the radio measurements contain near-sun plasma information on spatial and temporal scales that cannot be measured by other techniques.

**Comparison with Other Experiments**
Dedicated missions to study relativistic gravity have been proposed in the past (e.g., Bender et al., 1989) but recent simulations (Milani et al., 2002, Ashby et al., 2007) indicated that precision radio tracking of a spacecraft inserted in a circular orbit around Mercury could lead to an improvement in the determination of many PPN parameters. Testing relativistic gravity was recognized as a crucial scientific objective of BepiColombo at the inception of the project, although it is primarily a planetary mission. The Ka-band Transponder and accelerometer instruments will allow the MORE team to carry out many classical tests of relativity in the best dynamic solar system conditions. Testing general relativity, however, should not be viewed as a race between missions. Each experimental result, with important implications in physics and cosmology, would require confirmation by other missions and the science community benefits from multiple investigations in this field.

**Cassini**
Launched in 1997, the Cassini mission to Saturn had a long cruise period during which a relativity experiment was carried out during the solar conjunction period of 2002. Radio science data at X- and Ka-bands were acquired at the DSN’s station equipped for precision measurements at Ka- and X-bands for approximately one month centered on the solar conjunction where the minimum impact parameter was 1.6 solar radii, and no occultation by the Sun. The experiment was carried out using
Doppler (range-rate) observations, a method that had not been used until Cassini for such experiments because of the overwhelming noise contribution of the solar corona. Cassini overcame this limitation by augmenting the standard X-band link with a high frequency Ka-band link, providing a multi-frequency link system that calibrated and removed the effect of the solar corona. This capability was enabled on Cassini by a Ka-band Translator payload provided by ASI (different design from the MORE Ka-band Transponder also provided by ASI). The experiment determined, in agreement with general relativity (Bertotti et al., 2003), the parameter \( \gamma = 1 + (2.1 \pm 2.3) \times 10^{-5} \).

**Gravity Probe B**

Gravity Probe B (GPB), a polar orbiting spacecraft containing four very precise gyroscopes, sought direct measurement of the geodetic and frame-dragging precession of a gyroscope’s spin axis as the spacecraft orbits the spinning Earth. The GPB measurement can be cast in terms of PPN parameters. It is expected to determine \( \gamma \) with substantially better accuracy.

**MESSENGER**

The MESSENGER mission’s published science objectives (Solomon et al., 2001) do not include determination of PPN parameters or \( J_2 \) of the Sun. MESSENGER is equipped with only one radio link (X-band) so precision measurements would be severely degraded by the un-calibrated noise on Doppler and range from the solar corona. The absence of an accelerometer further limits the necessary precision calibration of non-gravitational forces acting on the spacecraft.

**Lunar Laser Ranging**

Williams et al. (2004) reported analysis of laser ranges to the Moon that provide increasingly stringent limits on any violation of the Equivalence Principle and also enable very accurate tests of relativistic gravity. They report a value for the Strong Equivalence Principle violation parameter \( \eta \) of \( (4.4 \pm 4.5) \times 10^{-4} \). Williams et al. (2004) also reported the search for a time variation in the gravitational constant results in \( \dot{G}/G = (4 \pm 9) \times 10^{-13} \) per year. Further measurements with improved accuracy in the next 5 to 10 years appear likely to improve the accuracies by factors of roughly 3 to 5.

**Gaia**

To be launched in 2011, Gaia will chart a three-dimensional map of our galaxy and provide unprecedented positional and radial velocity measurements with the accuracies needed to produce a stereoscopic and kinematic census of about one billion stars. Additional scientific products include a number of stringent new tests of general relativity and cosmology. Gaia is expected to provide a precision of \( \gamma \) of \( 1 \times 10^{-6} \) (Mignard 2009), an improvement approaching two orders-of-magnitude better than the current best estimate (Bertotti et al., 2003). Gaia, however, will not address the other MORE science objectives.

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Figure 2: MORE end-to-end instrumentation: two uplink frequencies (X-band and Ka-band) are transmitted simultaneously from a ground station and three coherent downlink signals are coherently returned back by the spacecraft.
Conclusion and Goals

The BepiColombo Mercury Orbiter Radio-science Experiment team will carry out high precision tests of relativistic gravity in the most desirable laboratory in the solar system, the gravitational field of the Sun. General relativity predicts definite values of the PPN parameters but alternate theories of gravity predict deviations from these values. Of the 10 parameters, 4 are considered for improvement by MORE techniques, namely the PPN parameters $\gamma$, $\beta$, $\eta$, and $\alpha$. In addition, the solar oblateness will be determined with much improved accuracy, useful information will be obtained on the possible rate of change of the gravitational constant, and properties of the solar corona will be monitored accurately. Testing relativistic gravity was recognized as a crucial scientific objective of BepiColombo; the Ka-band Transponder and accelerometer instruments enable many tests of relativity. Testing general relativity should not be viewed as a race between missions since experimental result, with important implications in physics and cosmology, would require confirmation by other missions and the science community benefits from multiple investigations in this field. With BepiColombo, comparable accuracies would be achieved with quite different types of measurements for $\gamma$, $\beta$, $\eta$, and $\dot{G}/G$, and the accuracy for $J2$ of the Sun would be improved by a substantial factor.

The goal of this white paper is inform the community of the planned experiments and seek continued support to enable them. The team intends to seek NASA support for participation in these important investigations with the range and range-rate radio tracking with the instrumentation of the Deep Space Network. A proposed advanced ranging instrument for dual X- and Ka-band wide band coherent links and expanded Ka-band uplink capability throughout the network would increase the sensitivity and experiment coverage.

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References

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