FY23 Topic Areas Research and Technology Development (TRTD)

Exploring quantum advantage in GRACE geopotential estimation

Principal Investigator: Alexandre Guillaume (398)**; Co-Investigators:** Dah-Ning Yuan (392)

Strategic Focus Area: Water and carbon cycles

Objectives

The main objective of this task was to apply a quantum algorithm designed for quantum gate computing to a JPL computing task, GRACE geopotential estimation. To this end, we chose to implement the Harrow-Hassidim-Lloyd (HHL) algorithm because it is a quantum algorithm that can solve linear system problems and provide exponential speedup over a classical solver [1]. The proposal listed two sub-objective to reach the main one:O1. Implement the HHL algorithm on different gate quantum computers accessed via AWS Braket. This objective partially allowed us to benchmark available quantum platforms.O2. Develop a roadmap of quantum linear solvers and how they could impact GRACE and successors. We proposed to explore in practice theoretical advances and alternative to the HHL algorithms. This second step should have allowed us to benchmark different quantum algorithms to solve linear problems and, more importantly for GRACE missions and increase the size of solved linear systems from 2 by 2 to 2^{17} by 2^{17}.

Background

Quantum computers can outperform conventional computers on certain tasks. Current implementations have typically up to 100 quantum bits (qubits), which is too small to perform most algorithms with error correction to compensate for qubits imperfections. Hence, the challenge in the noisy intermediate-scale quantum (NISQ) era we are in is to learn how to utilize today's solutions, albeit limited, to take advantage of this new technology as progress is made towards larger and more useful quantum computersDoubling the spatial resolution of a GRACE solution will directly increase the size of the solved-for problem by a factor of 4, which coupled with the cubically increasing complexity $\mathcal{O}(N^3)$ of classical inversion algorithms illustrates how ill-suited they are for scaling this class of problems (for an $N \times N$ matrix). Similar satellite gravimetry missions targeting other planetary bodies (e.g. GRAIL at the moon) have performed inversions of size 3,200,000 x 3,200,000; the quantum inversions performed here (with a complexity $\mathcal{O}(\log(N))$ for the HHL algorithm) have the potential to benefit similar future missions.

Approach and Results

We implemented the HHL algorithm and ran it on different quantum computers Rigetti Aspen-M-3, IonQ Aria-1/2, OQC's Lucy and Braket state vector simulator SV1. The experimental results (e.g. Figure 1) are imperfect and characteristic of a noisy quantum computer of the current NISQ era. They constitute an evaluation of the state-of-the-art, the starting point from which to improve both in quality and problem size using a series of HHL alternatives described in the proposal to reach the solution size and quality required by GRACE in practice.

We implemented a hybrid version of the HHL algorithm where a quantum part of the original HHL algorithm has been replaced by a computation on a conventional computer [2]. The main result from Figure 2. is that the hybrid HHL algorithm is more accurate than its fully quantum counterpart. Secondly, this experiment confirmed what we observed in the first experiment (HHL), namely, that the trapped-ions qubit systems of IonQ are substantially better than the superconducting circuits implementations (Rigetti and OQC).

$$|\phi\rangle_{14} = \frac{1}{4} \times \left(3\,|1000\rangle + |1001\rangle + \sqrt{3}\,|0000\rangle - \sqrt{3}\,|0001\rangle\right)$$

0000	0001	1000	1001
3/16	3/16	9/16	1/16
0.1875	0.1875	0.5625	0.0625

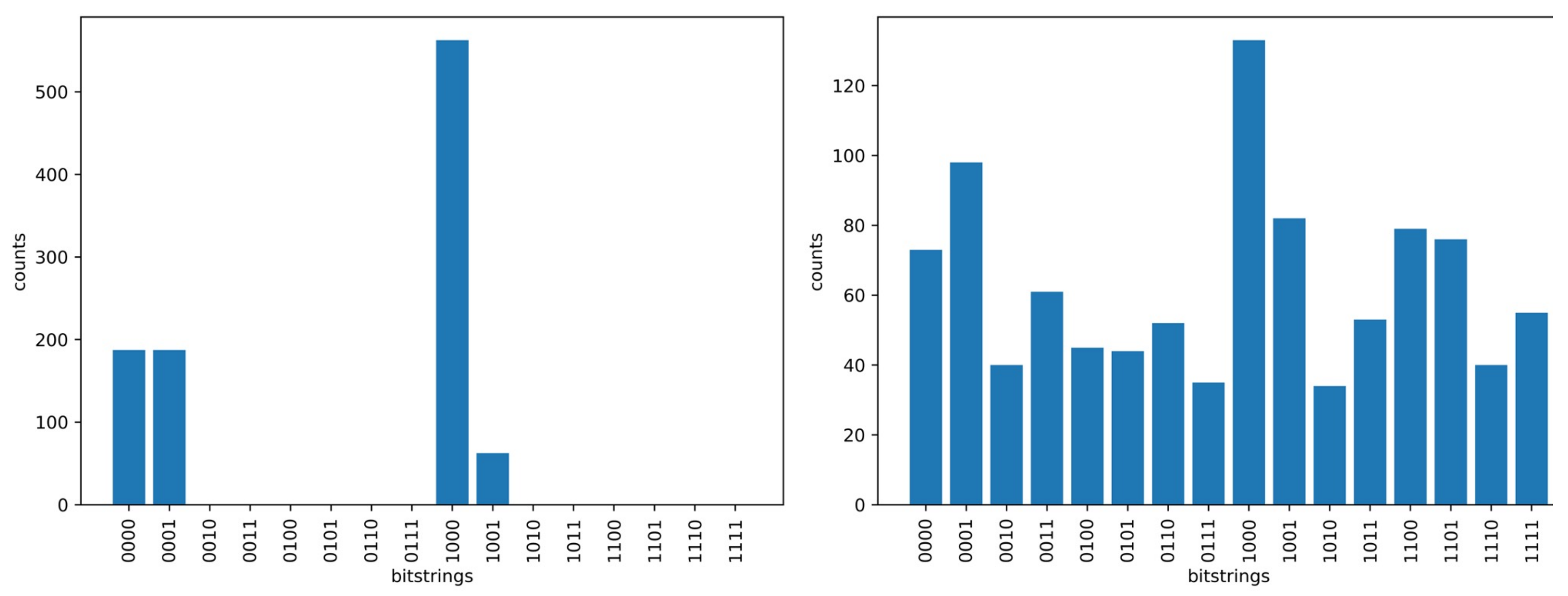

Figure 1. HHL expected probabilities (left) and measured probabilities using Rigetti Aspen-M-3 processor (right).

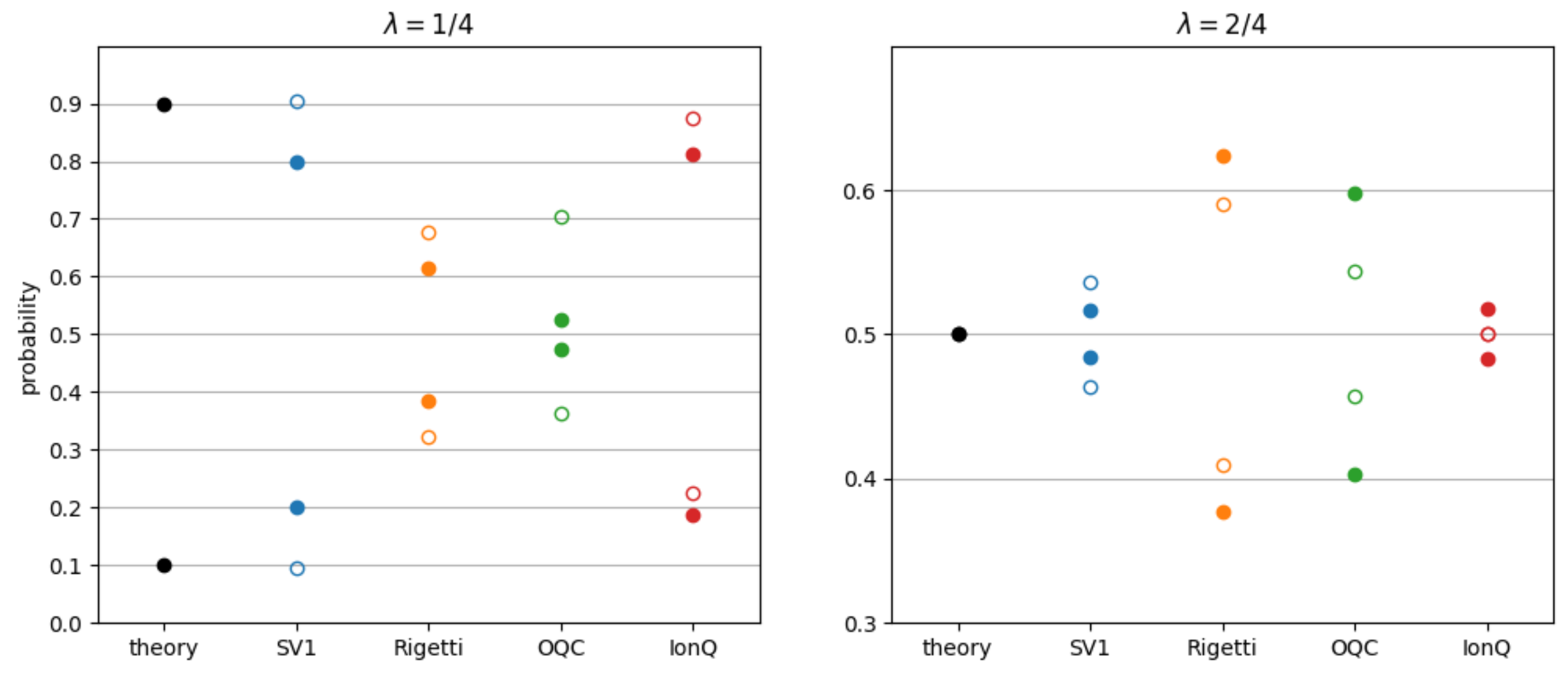

Figure 2. Solutions for HHL (full circle) and hybrid HHL (open circles) algorithms runs.

Significance/Benefits to JPL and NASA:

Both 9x and 8x program offices have been interested in exploring possible applications of quantum computing at JPL. The results showed at the mid-year review illustrated very well the state-of-the-art limitations of noisy intermediate-scale quantum (NISQ) systems. From this starting point, we proposed to increase the size (up to 2^{17}x2^{17}) and quality of the solutions by implementing a series of alternatives and improvement to the HHL algorithm. Our implementation of a hybrid HHL successfully showed an improvement over the HHL algorithm.
This effort started an expertise in quantum gate computing at JPL. This expertise will allow JPL to evaluate quantum computing solutions.

[1] Aram W. Harrow, Avinatan Hassidim, and Seth Lloyd. "Quantum algorithm for linear systems of equations," Phys. Rev. Lett. 103, 150502 (2009), https://doi.org/10.1103/PhysRevLett.103.150502.
[2] Yonghae Lee, Jaewoo Joo and Soojoon Lee, "Hybrid quantum linear equation algorithm and its experimental test on IBM Quantum experience," Sci. Rep. 9, 4778 (2019), https://doi.org/10.1038/s41598-019-41324-9.

National Aeronautics and Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

www.nasa.gov

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PI/Task Mgr. Contact Information:
Alexandre.Guillaume@jpl.nasa.gov
818-433-2704